Indian Statistical Institute, Bangalore Centre
B.Math. (I year): 2008 - 2009
Semester II : Midsemestral Examination
Probability Theory - II

6.3.2009 Time: $2\frac{1}{2}$ hrs. Max. Marks : 80

Note: The paper carries 85 marks. Any score above 80 will be treated as 80.

1. (15 marks) Let X and Y be independent random variables each having a uniform distribution over $(0,1)$. Find Prob. $(|X-Y| \leq 0 \cdot 4)$.

2. (10 marks) Let X and Y be independent random variables having exponential distribution with parameters λ_1 and λ_2 respectively. Find the distribution of $Z = \min\{X, Y\}$.

3. (5+5 marks) Let (X,Y) be a two dimensional absolutely continuous random variable such that X and Y have finite second moments.

   (i) Show that $E(XY)$ exists.

   (ii) Show that $Cov(X,Y) = E(XY) - E(X)E(Y)$.

4. (15+10 marks) Let $\alpha > 0, \beta > 0$ be fixed constants. Let

$$\begin{aligned} f(x,y) &= Cx^{\alpha}\exp\{-(\beta+y)x\}, \quad \text{if } x>0, y>0, \\ &= 0, \quad \text{otherwise.} \end{aligned}$$

where C is a constant.

   (i) Find C so that f is a probability density function on $\mathbb{R}^2$.

   (ii) Find the marginal probability density functions.

5. (17+8 marks) Let X and Y be independent $N(0,1)$ random variables. Let $V = 2X, W = X+Y$.

   (i) Show that (V,W) has a bivariate normal distribution, and find the covariance matrix.

   (ii) Find the conditional probability density function of W given $V = v$, for $v \in \mathbb{R}$.